

# A new solution to the puzzle of the long lifetime of $^{14}\text{C}$

D. Robson

*Department of Physics, Florida State University, Tallahassee, 32306, Florida, U.S.A.*

A new cluster model solution to the long standing nuclear structure problem of describing the anomalously long lifetime of  $^{14}\text{C}$  is presented. Related beta-decay data for  $^{14}\text{O}$  to states in  $^{14}\text{N}$ , gamma decay data between low lying positive parity states in  $^{14}\text{N}$  and the elastic and inelastic magnetic dipole electron scattering from  $^{14}\text{N}$  data are all shown to be very accurately described by the model. The shapes of the beta spectra for the  $A=14$  system are also well reproduced by the model. The model invokes four-nucleon tetrahedral symmetric spatial correlations arising from three- and four-nucleon interactions which yields a high degree of  $SU(4)$  singlet structure for the clusters and a tetrahedral intrinsic shape for the doubly magic  $^{16}\text{O}$  ground state. The large quadrupole moment of the  $^{14}\text{N}$  ground state is obtained here for the first time and arises because of the almost 100% d-wave deuteron-like-hole cluster structure inherent in the model.

## I. INTRODUCTION

The history surrounding the existing explanations of the  $^{14}\text{C}$  lifetime has involved many attempts [1–10], but none of them have been completely satisfactory. As noted in the most recent publications [10] a full understanding of all the data is expected to require three-nucleon interactions and/or clustering considerations. Unfortunately, until now, the details of clustering considerations and the nature of the three- (or more) nucleon interactions have not been addressed. Much of the previous work has involved  $A = 14$  wave functions based on two  $p$ -shell holes in the closed shell reference state (which is the  $^{16}\text{O}$  ground state with  $\{1s\}^4\{1p\}^{12}$  structure). This model is still the one used in the most recent structure publication [10]. Deviations from the simple two-hole shell model have [9] invoked additional multi-particle /multi-hole states but the calculations show poor convergence and do not provide any accurate description of the beta decay data. Other investigators [11, 12] chose to use phenomenological admixtures of the possible two-hole angular momentum configurations e.g., in  $L$ - $S$  coupling:  $\{^3S_1, ^1P_1, ^3D_1\}$  for the  $^{14}\text{N}$   $J^P = 1^+, T = 0$  ground state and  $\{^1S_0, ^3P_0\}$  for the  $J^P = 0^+, T = 1$  isospin states in  $^{14}\text{C}$ ,  $^{14}\text{N}$  and  $^{14}\text{O}$ . These type of attempts were criticized [8, 13] for being inconsistent with the conventional strong j-j coupling shell model. As we shall show the phenomenological approach appears to be closer to the model used here and with more searching might have resulted in the wave function admixtures presented here.

In view of the failure of the shell model approach to provide a complete description of the beta decay and allied data in the  $A = 14$  nuclei it does appear that the shell model approach is *not the optimal choice* as the basic picture for the  $^{14}\text{C}$  beta decay problem. That this appears to be the reality of the situation led this investigator to invoke a more realistic model of the *closed shell* nucleus  $^{16}\text{O}$  which includes multi-nucleon correlations. Such a correlated model has a longer history than the shell model as it dates back to Wheeler [14] in 1937. This was followed by other investigators [15, 16] and for  $^{16}\text{O}$  the alpha-particle cluster model relying on the similarity with the methane molecule  $\text{CH}_4$  was used to describe the energy levels as rotational vibrational states of a tetrahedral molecule in which the H-atoms were replaced by alpha-particles and the C-atom at the center was eliminated. All of the early work and almost all of the later efforts with the alpha-particle model (see reference [17] for a review) have assumed the alpha clusters to be uncorrelated  $\{1s\}^4$  configurations as in the simple spherical shell model. The lack of correlations within each cluster and the assumption of spherical intrinsic states leads to difficulties in obtaining accurate predictions with the spherical alpha particle cluster model. In particular we know of no attempts to describe the lifetimes of  $^{14}\text{C}$  or  $^{14}\text{O}$  using such models.

Quite early in the history of clustering this investigator proposed a model [18] which synthesized the simple cluster model with the shell model by introducing quark degrees of freedom into the bound states of nuclei with  $A = 2, 3$  and 4. The initial work (summarized in [19]) emphasized that many-body forces were to be expected and that the three and four nucleon bound states would have spatial correlations corresponding to point group symmetries  $D_{3h}$  and  $T_d$  for  $A = 3$  and 4 respectively. A specific model [20] for the two-nucleon systems based on quark dynamics and one pion exchange gave a realistic description of the deuteron and the phase-shifts for low partial waves. The parameters of the model are consistent with the one nucleon non-relativistic quark model and the basic symmetries of QCD and chiral symmetry are adhered to. This approach was then extended consistently [21] to the spin independent part of the three-nucleon interaction which showed that an *equilateral triangle* configuration with  $D_{3h}$  point group symmetry for the nucleons was strongly favored. The strong repulsive interactions of up to 2 GeV between each pair of nucleons leads [19] to a hole in the charge density distribution at the nuclear center as originally suggested by the authors of the experimental work [22].

The  $^4\text{He}$  ground state is expected to have *tetrahedral intrinsic spatial* symmetry since it maximizes the three-

nucleon triangular configurations occurring on the four equivalent faces of the tetrahedron. Indeed as indicated in [19] the elastic electron scattering data for  $^3\text{He}$  and  $^4\text{He}$  are very well described with equilibrium radii corresponding to triangular and tetrahedral geometry. Again the hole at the center of the  $^4\text{He}$  charge distribution is well described by the  $T_d$  model. Such spatial correlations are totally symmetric representations of the orbital angular momentum rotation group  $O(3)$  provided that the intrinsic configuration is rotated through the three Euler angles with equal weight and no parity change under inversion. These spatial point group symmetries lead automatically to totally antisymmetric  $SU(4)$  states for spin  $S$  and isospin  $T$ , i.e.,  $S = 1/2 = T$  for  $A = 3$ , and  $S = 0 = T$  for the alpha particle. The  $^3\text{He}$ ,  $^3\text{H}$  ground states are then simple one nucleon-hole states in the alpha-particle. Here we focus on the alpha particle which has  $J^P = 0^+$  and consequently  $L = 0$  only for its total orbital angular momentum (as also occurs in the simple  $\{1s\}^4$  shell model configuration). At this point we note that the quark model discussed above showed significant quenching of the one-pion exchange tensor interaction as the nucleon-nucleon separation distance decreased and for heavier meson exchanges between quarks there was essentially no interaction. The  $T_d$  spatial symmetry of the alpha-particle intrinsic state in this model is therefore expected to lead to very weak spin dependent contributions to the ground state cluster wave function. In what follows we will assume only the leading  $L = 0 = S = T$  state to be present in the ground state of  $^4\text{He}$  and in the intrinsic states of embedded four nucleon alpha-like clusters in  $^{16}\text{O}$ .

## II. BASIS WAVE FUNCTIONS

The cluster model wave functions used here show a strong resemblance to the shell model basis states and indeed for  $A = 14$  the two-hole states have the same total orbital ( $L$ ), spin ( $S$ ) and total angular momentum  $J$  as those used in shell model states. For comparison purposes we use the same notation as that of Genz *et al* [11] and in  $L$ - $S$  coupling the most general wave functions are

$$|^{14}\text{N}, J^P = 1^+, T = 0\rangle = \alpha \ ^3S_1 + \beta \ ^1P_1 + \gamma \ ^3D_1 \quad (1)$$

$$|^{14}\text{N}^*, J^P = 0^+, T = 1\rangle = \xi_0 \ ^1S_0 + \eta_0 \ ^3P_0 \quad (2)$$

$$|^{14}\text{C}, J^P = 0^+, T = 1\rangle = \xi_- \ ^1S_0 + \eta_- \ ^3P_0 \quad (3)$$

$$|^{14}\text{O}, J^P = 0^+, T = 1\rangle = \xi_+ \ ^1S_0 + \eta_+ \ ^3P_0 \quad (4)$$

wherein only the  $L^P = 1^+$  (denoted by  $P$ ) states have the same angular momentum substructure as the shell model uncorrelated  $p^{-2}$  hole states. In what follows the angular momentum substructure of the S and D cluster model states are not the same as the shell model uncorrelated  $p^{-2}$  hole states. The normalization of the above four configurations however is the same, i.e.,

$$\alpha^2 + \beta^2 + \gamma^2 = \xi_\iota^2 + \eta_\iota^2 = 1 \quad (5)$$

with  $\iota = 0, +$  and  $-$ . The values of  $\xi_\iota, \eta_\iota$  allow for a possible isospin triplet symmetry breaking which is needed to describe the difference in the  $\log(f_{At})$  values for the  $\beta^+$  and  $\beta^-$  decays of  $^{14}\text{O}$  and  $^{14}\text{C}$  leading to the ground state of  $^{14}\text{N}$  respectively.

The reference state for the cluster model hole states is also the  $J^P = 0^+, T = 0$  ground state of  $^{16}\text{O}$ , but here we assume the reference state is highly correlated with four alpha-like intrinsic clusters with their centers of mass having equilibrium points at the four corners of a tetrahedron as in Fig. 9 of [19]. As in  $^4\text{He}$  the ground state of  $^{16}\text{O}$  is found by rotating over all Euler angles with equal weight and requiring no change of parity under inversion of all the spatial coordinates. The strong cluster correlations between nucleons in the same alpha-like intrinsic cluster and the much weaker correlations between two nucleons in different intrinsic clusters leads to the assignment of  $S$ - and  $D$ -states for two nucleons taken from the same cluster and  $P$ -states when the two nucleons are taken from different clusters. In the cluster model presented here the individual clusters have  $SU(4)$  singlet structure and for four clusters satisfying identical boson symmetry then the  $^{16}\text{O}$  reference state will be pure  $L$ - $S$  coupled. Removing a pair from an individual cluster allows the pair to have antisymmetric  $SU(4)$  quantum numbers, which yields only  $(S = 0, T = 1)$  and  $(S = 1, T = 0)$   $SU(4)$  hole states. This in turn requires the relative motion of the two nucleons to be in an even parity spherical harmonic  $Y_\ell$  state. As discussed earlier the  $\ell$ -value of the relative motion within a tetrahedral cluster is taken here to be zero. The total orbital angular momentum can be  $L = 0$  or 2 if the non-zero contribution comes from the motion of the center of mass of the two nucleons relative to the core. However

when the two nucleons are taken from different clusters the allowed  $SU(4)$  pair states must be symmetric in order to satisfy the bose permutation symmetry between the two identical clusters. In this case the allowed  $SU(4)$  states are  $(S = 0, T = 0)$  and  $(S = 1, T = 1)$  which requires their relative motion orbital angular momentum to be odd valued. The ground states of the mirror nuclei  $^{15}\text{N}$  and  $^{15}\text{O}$  have  $J^P = 1/2^-$  and as in the shell model they are represented by proton and neutron  $p_{1/2}$ -hole states in the ground state of  $^{16}\text{O}$  respectively. In the shell model the two p-holes can only have antisymmetric spatial states if  $L = 1$  and this also holds true for the cluster model. Consequently we assign the  $P$  configurations to cluster model configurations in which individual p-holes are taken in different clusters. For the  $T = 1$  states in  $A = 14$  there are only two basis states: (a) the  $^1S_0$  state involving a pure  $L = 0$  dinucleon cluster extracted from the same alpha-like cluster and (b) the  $^3P_0$  state involving a pure  $L = 1$  with each p-hole taken from a different alpha-like cluster. Similarly for the  $T = 0$  basis states in  $^{14}\text{N}$  there are two types of basis states: (a) the  $^3S_1$  and  $^3D_1$  states are pure  $L = 0$  and 2 deuteron-like clusters extracted from the same alpha-like cluster and (b) the  $^1P_1$  state with  $L = 1$  only and with each p-hole coming from a different alpha cluster.

The foregoing is important because it matters when considering the energy matrix for the individual systems. It also matters when considering observables that are dependent on the angular momentum substructure of the orbital  $L$ -states or on their radial wave functions. This is vital in the case of the quadrupole moment of the ground state of  $^{14}\text{N}$  and in describing elastic and inelastic electron scattering data. The  $\beta$ -decay data,  $M1$  gamma transitions in  $^{14}\text{N}$  and the magnetic moment of the ground state of  $^{14}\text{N}$  are independent of the substructures discussed above as these observables depend only on the admixture amplitudes defined in eqs. (1-5).

### III. ENERGY MATRICES

For  $T = 1$  states with two basis states the  $2 \times 2$  matrices involve the coupling between the  $^1S_0$  and  $^3P_0$  states which require a spin-orbit interaction which is antisymmetric in spin space and also in orbital space. Such an interaction can be constructed from the sum of one-body operators for each nucleon and is a superposition of the nuclear spin-orbit  $V_{N_{so}}$  and the electromagnetic spin-orbit  $VE_{so}$ , see [23]. Specifically these interactions for each nucleon are

$$VN_{so}(\text{neutron}) = \boldsymbol{\sigma}_N \cdot \mathbf{grad}\{UN(\mathbf{r}_N)\} \times \mathbf{p}_N = VN_{so}(\text{proton}) \quad (6)$$

$$VE_{so}(\text{neutron}) = [\boldsymbol{\sigma}_N \cdot \mathbf{grad}\{UE(\mathbf{r}_N)\} \times \mathbf{p}_N]\mu_n \quad (7)$$

$$VE_{so}(\text{proton}) = [\boldsymbol{\sigma}_N \cdot \mathbf{grad}\{UE(\mathbf{r}_N)\} \times \mathbf{p}_N](\mu_p - 1/2). \quad (8)$$

In these generalized spin-orbit interactions we assume that  $UN(\mathbf{r})$  and  $UE(\mathbf{r})$  are the nuclear and Coulomb potentials which can involve tetrahedral harmonics with orbital-values of 0, 3, 4, 6 etc. For convenience the nuclear terms in eq. (6) are taken to be the same for neutrons and protons and any differences (which should exist) are taken to be included in the overall magnitude of the electromagnetic spin-orbit terms. The matrix elements for the  $T = 1$   $2 \times 2$  matrix we denote by  $H_{SS}^1$ ,  $H_{PP}^1$  and  $V_{SP}^1 (= V_{PS}^1)$ , wherein the  $S$  and  $P$  subscripts imply the  $^1S_0$  and  $^3P_0$  basis states. Values for these matrix elements are different for  $^{14}\text{C}$ ,  $^{14}\text{N}$  ( $T = 1$ ) and  $^{14}\text{O}$  and for diagonal elements one has the “unperturbed” energies of the system whereas the off-diagonal elements are simply the matrix elements of the spin-orbit interactions given above. The  $V_{SP}^1$  matrix elements are charge dependent because the nucleon magnetic moments  $\mu_n$  and  $\mu_p$  are of opposite sign. We characterize  $V_{SP}^1$  for each member of the isospin triplet for the nuclear spin-orbit matrix element by  $v_{nuc}$  and the strength for the electromagnetic spin-orbit for two proton holes by  $v_{el}$ . Consequently the values of the matrix elements  $V_{SP}^1$  for the isospin triplet are given by  $v_{nuc} + f(N, N)v_{el}$  with  $f(p, p) = 1$ ,  $f(n, n) = \mu_n/(\mu_p - 1/2)$ ,  $f(p, n) = (f(p, p) + f(n, n))/2$ . The amplitudes  $\xi_\iota, \eta_\iota$  for the three values of  $\iota = -, 0$  and  $+$ , corresponding to  $^{14}\text{C}$ ,  $^{14}\text{N}^*$  and  $^{14}\text{O}$  respectively, are found by diagonalizing the three  $2 \times 2$  matrices when specific values of the unperturbed energy differences  $H_{PP}^1 - H_{SS}^1$ ,  $v_{nuc}$  and  $v_{el}$  are used to fit the experimental data. In particular the experimental values of the three energy differences between the ground states of the isospin triplet and the corresponding first excited states with  $J^P = 0^+, T = 1$  are constraints to be satisfied by the eigenvalues of the three diagonalizations. These energy spacings are 6.589 MeV in  $^{14}\text{C}$ , 6.305 MeV in  $^{14}\text{N}^*$  and 5.91 MeV in  $^{14}\text{O}$ . This leaves two parameters out of the five input parameters for the  $T = 1$  sector to be determined.

For the  $J^P = 1^+, T = 0$  case in  $^{14}\text{N}$  there are three basis states which can be labeled by  $D, S$  and  $P$  corresponding to  $^3D_1$ ,  $^3S_1$ , and  $^1P_1$ . We expect the  $D$ -state to be the least bound state in the cluster model as it is even in all shell models used historically. In the cluster model we expect after diagonalization that the  $D$ -state occupation will be close to 100% and will be the ground state of  $^{14}\text{N}$ . The other two eigenstates should be the  $1^+$  states at 3.948 MeV and 6.204 MeV respectively. The diagonal matrix elements of the  $3 \times 3$  energy matrix will have two unperturbed energy

input values e.g.,  $H_{SS}^0 - H_{DD}^0$  and  $H_{PP}^0 - H_{DD}^0$  and after diagonalization the eigenvalue of the system should have the energy splittings of 3.948 MeV and 6.204 MeV respectively. The other three input values are the matrix elements  $V_{DP}^0$ ,  $V_{SP}^0$  (which are non-zero from the spin-orbit interaction in a similar manner to the  $T = 1$ ,  $V_{SP}^1$  discussed above) and  $V_{DS}^0$  with the latter expected to be very weak since it can only arise from a scalar product of rank two tensors in spin and orbital spaces. This expectation is based on the fact that tetrahedral symmetry does not have quadrupole harmonics since this suppresses all tensor interactions in the  $^{16}\text{O}$  system. The  $T = 0$  sector (like the  $T = 1$  sector) involves five input parameters and fits to the energy differences reduces this number of parameters to three which must be determined from other data. Thus overall we have five variables in the energetics, two from the  $T = 1$  sector and three from the  $T = 0$  sector, and at first sight these could be determined by the three  $\beta$ -decay data sets for  $^{14}\text{C}(\beta^-)$  and  $^{14}\text{O}(\beta^+)$  going to the ground and first excited  $J^P = 1^+$  states in  $^{14}\text{N}$ . The five observables are the three  $\log(f_A t)$  values and the shapes of the  $\beta^-$  and  $\beta^+$  decays to the  $^{14}\text{N}$  ground state. The shape of the  $\beta^+$  decay to the 3.948 MeV state in  $^{14}\text{N}$  is not measured and in any event is expected to be constant as this transition has a  $\log(f_A t)$  of 3.138 corresponding to an unhindered Gamow-Teller transition. Unfortunately, as shown by Towner and Hardy [24], one needs to invoke renormalized axial ( $g_A$ ) and magnetic ( $g_l$  and  $g_s$ ) couplings to obtain the correct results for the strongly hindered  $\beta$ -decay data and the radiative width of the  $T = 1$   $^{14}\text{N}$  (2.313 MeV) state.

#### IV. RENORMALIZED OPERATORS

The operators needed in the remainder of this paper are the free nucleon coupling constants  $g_A, g_{lp}, g_{ln}, g_{sp}$  and  $g_{sn}$  corresponding to the axial vector ( $g_A = 1.2695$ ), the orbital  $g$ -factors ( $g_{lp} = 1$  and  $g_{ln} = 0$ ) and spin  $g$ -factors ( $g_{sp}/2 = 2.79285$  and  $g_{sn}/2 = -1.91304$ ), with all the magnetic couplings being in units of nuclear magnetons (n.m.). The renormalized  $g_A$  for the  $\beta$ -decay studies in  $A = 14$  is taken from the  $^{15}\text{O}(\beta^+)^{15}\text{N}$  mirror state transition as suggested by Towner and Hardy. The transition is assumed to take place between single p-shell holes in the  $^{16}\text{O}$  reference state and results in a renormalized value of  $g_A^* = 1.0885 = (g_A - 0.181)$  which yields the  $\log(f_A t) = 3.644$  for the Gamow-Teller component [25] of the  $^{15}\text{O}(\beta^+)^{15}\text{N}$  transition. The renormalized magnetic operators are taken here initially to fit the magnetic moments of the ground states of the mirror nuclei  $^{15}\text{N}$  and  $^{15}\text{O}$  based on a single p-orbital-hole in the  $^{16}\text{O}$  ground state. In looking at these magnetic moments using the simple formulas for a p-hole with  $j^P = 1/2^-$  given by

$$\mu_i = 1/3(2g_{li} - g_{si}/2) = g_{ji}/2 \quad (9)$$

where for  $i$ =free neutron  $g_{nl} = 0$  and for a free proton  $g_{pl} = 1$  and similarly  $g_{ns}/2 = -1.91304$  n.m. and  $g_{ps}/2 = 2.79285$  n.m. one finds values of the moments  $g_{ji}/2$  as -0.26428 n.m. for  $^{15}\text{N}$  and +0.63768 n.m. for  $^{15}\text{O}$ . These are not in good agreement with the experimental values of -0.28319 n.m. for  $^{15}\text{N}$  and +0.7189 n.m. for  $^{15}\text{O}$ . It is necessary to use renormalized magnetic couplings as pointed out by Towner and Hardy [24]. They obtained these by including bound state shell model corrections arising from core polarization and meson-exchange currents.

In the cluster approach the p-hole arises by breaking the individual alpha-like clusters in which it is embedded. This suggests that an initial guess for the renormalized  $g_{ji}^*/2$  values in the  $A=15$  systems should be given by

$$g_{ji}^*(A = 15)/2 = g_{ji}(A = 15)/2 \times \{g_{ji}^*(A = 3)/g_{ji}(A = 3)\} \quad (10)$$

in which  $g_{ji}^*(A = 3)/2$  are the observed moments for the s-hole states in a free alpha particle which are -2.12750 n.m. and +2.97896 n.m. for the neutron and proton s-holes respectively. The values of  $g_{ji}(A = 3)/2$  are taken to be the free nucleon magnetic moments. The renormalized magnetic moments  $\mu_i^*$  for  $A = 15$  are found to be -0.28189 n.m. and +0.7092 n.m. for  $^{15}\text{N}$  and  $^{15}\text{O}$  respectively. This cluster model approach to the renormalization of the magnetic moments of the nucleon-holes is remarkably accurate in obtaining values of the  $A = 15$  magnetic moments which are within 0.5% and 1.4% of the observed values for  $^{15}\text{N}$  and  $^{15}\text{O}$  respectively.

The above discussion for finding renormalized magnetic coupling constants that describe the  $A = 15$  mirror states appears to be another validation of the alpha-like cluster model for the  $^{16}\text{O}$  reference state. However further modifications to the values of  $g_{li}^*$  and  $g_{si}^*$  are needed to obtain a consistent and completely accurate description of the magnetic moment data for the ground state of  $^{14}\text{N}$  as well as the  $A=15$  mirror pair states. In using the result from (10) above we infer that  $g_{lp}^*$  is approximately 1.1,  $g_{ln}^*$  is approximately 0.0,  $\mu_{sp}^*$  is approximately 3.0 and  $\mu_{sn}^*$  is close to -2.15 (or if  $g_{ln}^* = 0$  then  $\mu_{sn}^* = -2.1567$  so that the magnetic moment of  $^{15}\text{O}$  is exactly reproduced). Best results are achieved with the values:  $g_{lp}^* = 1.112$ ,  $g_{ln}^* = 0$ ,  $\mu_{sp}^* = +3.0735664$  and  $\mu_{sn}^* = -2.1567$  using the wavefunctions that fit the  $\beta$ -decay data as discussed below.

Not only does this set of renormalized couplings fit the magnetic moments of  $^{14}\text{N}$  ( $\mu = +0.403761$  n.m.),  $^{15}\text{N}$  and  $^{15}\text{O}$  but also exactly fits the magnetic moment of the first  $J^P = 3^-$  state in  $^{16}\text{O}$ . The measured value for this  $3^-$  state is  $\mu = +1.668$  n.m. corresponding to  $g_J$  with the isoscalar gyromagnetic  $g = +0.556$  (error is .005). In the

tetrahedral model this  $3^-$  state is a collective rotational excitation of the  $0^+$  ground state of  $^{16}\text{O}$  for which the  $g$  factor is  $Z/A = 1/2$  if the orbital  $g_{li}$  is 1 for protons and 0 for neutrons. Using the cluster renormalized orbital  $g$ -factors given above yields  $g = g_{lp}^*/2 = +0.556$  in perfect agreement with the data. In obtaining the magnetic moment for the ground state of  $^{14}\text{N}$  the shell model formula [11]

$$\mu = 2^{-1/2}\{(\mu_p + \mu_n)W_1/2^{-1/2} + W_3\} \quad (11)$$

(in which  $W_1, W_3$  are  $(2\alpha^2 - \gamma^2)$ ,  $(\beta^2 + 3\gamma^2/2)/2^{-1/2}$  respectively), is modified for the cluster model to  $\mu^*$  given by

$$\mu^* = 2^{-1/2}\{\mu_d^*W_1/2^{-1/2} + g_{lp}^*W_3\} \quad (12)$$

with  $\mu_d^*$  being the renormalized isoscalar magnetic moment of the deuteron-like hole state in  $^{14}\text{N}$ . The deuteron-hole can involve internal orbital angular momentum of  $l = 0$  and 2 due to the two-body tensor interaction between the neutron- and proton-holes in the same alpha-like cluster. We use the equivalent parameterizations for  $\mu_d^* = f_d(\mu_p + \mu_n)$  or  $\mu_d^* = f_d^*(\mu_p^* + \mu_n^*)$  the only new parameter is  $f_d$  (or equivalently  $f_d^*$ ) since we use the renormalized nucleon-hole magnetic moments given above. If  $f_d^*$  is unity then the probability  $P_D$  of any  $l = 2$  state in the deuteron-hole is zero. For a small value of  $P_D$  we should have  $f_d^*$  being slightly less than unity. Using the renormalized operators which fit the  $A = 15$  and  $A = 16$  magnetic moments then the  $^{14}\text{N}$  moment given by (12) is 0.382 n.m. if  $f_d^* = 1$  and is 0.403761 n.m. when we choose  $f_d^* = .950706$  (or  $f_d = .990755$ ) which is consistent with a small value for  $P_D$ . The value of  $P_D = 8.3\%$  obtained here using

$$P_D = (2/3)(1 - f_d^*)(\mu_p^* + \mu_n^*)/(\mu_p^* + \mu_n^* - g_{lp}^*/2) \quad (13)$$

is not inconsistent with any information concerning  $D$ -states in the alpha particle. However by allowing for 1% errors in the magnetic moments of  $^{15}\text{O}$  and  $^{16}\text{O}$  one can obtain fits to the magnetic moment of  $^{14}\text{N}$  which have values of  $P_D$  as low as 4.7%.

## V. RESULTS FOR BETA-DECAY OBSERVABLES

For Gamow-Teller (GT) transitions in the  $A = 14$  nuclei we restrict our considerations to  $J^P = 0^+$  initial states leading to  $J^P = 1^+$  states in  $^{14}\text{N}$ . The GT matrix element is given by [24]

$$\text{MGT} = g_A^* 6^{1/2} (\xi_l \alpha - \eta_l \beta / 3^{1/2}) \quad (14)$$

where we include the renormalized axial coupling  $g_A^*$  in the definition of MGT. With this definition the  $f_{At}$  is given by [24]

$$f_{At} = 6146/|\text{MGT}|^2 \text{s}. \quad (15)$$

in which the  $f_A$  rate functions are corrected to include the effects of the nuclear structures via the shape function  $C(Z, W)$  used by Towner and Hardy

$$C(Z, W) = |\text{MGT}|^2 k(1 + aW + \mu_1 \gamma_1 b/W + cW^2) \quad (16)$$

following the format of Genz *et al* that involves the  $a, b, c$  variables which are dependent on the details of the nuclear structure model. To the accuracy needed for  $A = 14$  beta-decay shapes we use  $\mu_1 \gamma_1 = 1$ . The parameter  $k$  is used to fit the data and  $W$  is the total electron (positron) energy in units of the electron rest mass energy. The data [27] for the  $^{14}\text{C}(\beta^-)^{14}\text{N}$  decay gives information on the slope parameter ( $a$ ) whereas the data [28] for the  $^{14}\text{O}(\beta^+)^{14}\text{N}$  decay gives information on all three parameters ( $a, b$  and  $c$ ). The expressions for  $a, b$  and  $c$  as well as the nuclear matrix elements are given in detail in Genz *et al* [11] and also in Garcia and Brown [26] who noted that the term denoted by  $V_4$  has the opposite sign from that given by Genz *et al*. Our calculations use the sign choice of Garcia and Brown for  $V_4$  as it is consistent with the relation between 2-hole states and 2-particle states. We choose not to include these lengthy relationships as they are readily available in [11, 26]. It is important to note that the shape functions calculated using the formulas in Genz *et al* have some terms which are model dependent. In particular we have used renormalized  $g^*$  for axial and magnetic couplings in place of the free nucleon  $g$ 's and also replaced the oscillator length ( $b = 1.7\text{fm}$  in Genz *et al*) by the cluster model  $b_c$  value obtained from the cluster model fit (using linear combinations of p-state radial harmonic oscillator states) to the inelastic electron scattering data as discussed in VI below. The shape function is sensitive to the choice of  $b_c$ . The behavior of the slope parameters  $a_-$  and  $a_+$  for

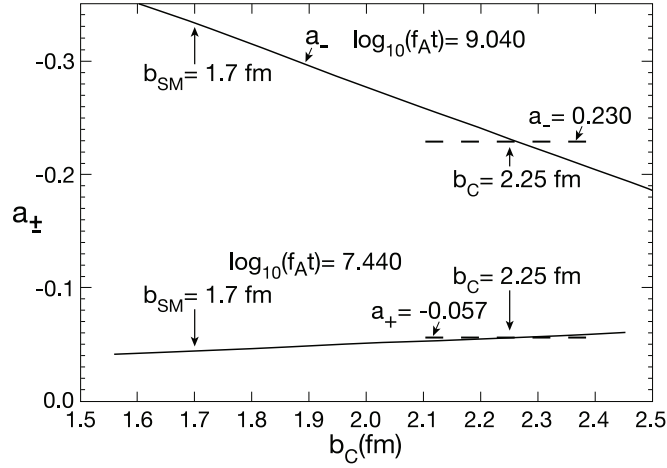


Figure 1: The slope parameters  $a_-$  and  $a_+$  used in (16) for  $C(Z, W)$  for the  $^{14}\text{C}(\beta^-)$  and  $^{14}\text{O}(\beta^+)$  decays to the ground state of  $^{14}\text{N}$  respectively as a function of the average oscillator length  $b_c$ . The  $\log(f_A t)$  values are independent of  $b_c$  and correspond to the tabulated wavefunction admixtures given in section V.

$^{14}\text{C}(\beta^-)$  and  $^{14}\text{O}(\beta^+)$  decaying to the  $^{14}\text{N}$  ground state using the wavefunctions shown below are shown in Fig. 1 as a function of the average oscillator parameter which is denoted by  $b_c$  in Fermi units.

The cluster model wavefunctions that fit all the data in terms of the coefficients  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\xi_\iota$ ,  $\eta_\iota$  are given for  $T = 0, J^P = 1^+$  and for  $T = 1, J^P = 1^+$  in Table I and Table II respectively.

	$\alpha$	$\beta$	$\gamma$
$ ^{14}\text{N}, E = 0\rangle$	.0169003	.1860092	.9824026
$ ^{14}\text{N}, E = 3.948\rangle$	.7600690	-.6407632	.1082473
$ ^{14}\text{N}, E = 6.204\rangle$	.6496225	+.7448645	-.1522089

Table I: Numerical values of  $\alpha, \beta, \gamma$  for the three  $1^+$  states in  $^{14}\text{N}$  which result from the diagonalization of the symmetric  $3 \times 3$  matrix as discussed in the text.

	$\xi_\iota$	$\eta_\iota$
$ ^{14}\text{C}, E = 0\rangle$	.9865400	.1635201
$ ^{14}\text{N}, E = 2.313\rangle$	.9909549	.1341954
$ ^{14}\text{O}, E = 0\rangle$	.9945400	.1043556
$ ^{14}\text{C}, E = 6.589\rangle$	-.1635201	.9865400
$ ^{14}\text{N}, E = 8.616\rangle$	-.1341954	.9909549
$ ^{14}\text{O}, E = 5.910\rangle$	-.1043556	.9945400

Table II: Numerical values of  $\xi_\iota, \eta_\iota$  for the two  $0^+$  states in each of the isospin triplet of  $A = 14$  nuclei which result from the diagonalization of the symmetric  $2 \times 2$  matrix as discussed in the text.

These structure coefficients are the eigenstates obtained by diagonalizing the symmetric  $3 \times 3$  matrix for  $T = 0$  with off-diagonal matrix elements (in MeV) :  $V_{SD}^0 = -0.28864$ ,  $V_{PD}^0 = -0.9772$ ,  $V_{SP}^0 = 1.079353$  and diagonal unperturbed energies  $E_{SS} = 4.7088$ ,  $E_{PP} = 4.873$  and  $E_{DD} = 0$ . Similarly there are three symmetric  $2 \times 2$  matrices for the  $T = 1$  systems and their matrices have elements given by Table III in which the unperturbed energies have an average value for the isospin triplet which is 6.03 MeV, that is remarkably similar to the energy splitting of the first two  $J^P = 0^+$  states in the reference system of  $^{16}\text{O}$ .

	$V_{SP}^1$	$E_{SS}$	$E_{PP}$
$^{14}\text{C}$	-1.062993	0	6.237
$^{14}\text{N}$	-.8381848	0	6.076
$^{14}\text{O}$	-.6133766	0	5.7813

Table III: Numerical values of the elements of the  $2 \times 2$  energy matrix for the isospin triplet of  $A = 14$  nuclei.

The major effect of the electromagnetic spin-orbit in the  $T = 1$  isospin triplet states lies in the relatively large differences of the  $\eta_i$  coefficients. In particular the magnitude of  $\eta_+$  for the  $^{14}\text{O}$  ground state is about 36% smaller than  $\eta_-$  for the mirror state in  $^{14}\text{C}$ . Indeed it is this large difference in the cluster model  $\eta$  coefficients which leads to an explanation of the large ratio of the MGT elements for  $^{14}\text{O}(\beta^+)/^{14}\text{C}(\beta^-)$ . It is vital to understand that it is the cluster correlations between the nucleons that damps the nuclear spin-orbit ( $v_{\text{nuc}}$ ) and enhances the electromagnetic spin-orbit ( $v_{\text{el}}$ ). The values used here are  $v_{\text{nuc}} = -0.817874$  MeV and  $v_{\text{el}} = -0.245119$  MeV. As defined above  $v_{\text{el}}$  is appropriate for the two proton-hole  $^{14}\text{C}$  system and the values of  $V_{SP}^1$  are given using the superpositions of  $v_{\text{nuc}} + f(N, N)v_{\text{el}}$  as given in **III** above. Although the symmetry breaking in  $\eta_+$  and  $\eta_-$  is large the overlap of the  $^{14}\text{C}$  and  $^{14}\text{O}$  ground states is 0.99822 which involves an overall symmetry breaking of less than 0.2%.

Using the above wave functions and  $b_c = 2.25$  fm leads to the values of MGT,  $\log(f_A t)$  and the shape parameters  $a, b, c$  given in Table IV.

Model	MGT	$\log(f_A t)$	$k$	$a$	$b$	$c$
$^{14}\text{O}(\beta^+)^{14}\text{N}(E = 0)$						
Cluster	0.01493	7.440	1.600	-0.056	0.033	0.002
Cluster	0.01373	7.513	1.990	-0.060	0.036	0.003
PBWT	0.01480	7.448	1.605	-0.054	0.035	0.002
Expt: $^{14}\text{O}$	0.018(0)	7.284(7)				
$^{14}\text{O}(\beta^+)^{14}\text{N}(E = 3.948)$						
Cluster	2.119	3.136	1.0	0.0	0.0	0.0
Expt: $^{14}\text{O}$	2.119(39)	3.138(16)				
$^{14}\text{C}(\beta^-)^{14}\text{N}(E = 0)$						
Cluster	-0.00237	9.040	-	-0.231	0.125	0.023
Cluster	-0.00343	8.718	-	-0.176	0.063	-0.003
PBWT	-0.00343	8.718	0.607	-0.235	0.013	0.013
Expt: $^{14}\text{C}$	0.002(0)	9.040(3)		-0.23(2)	(100-160 keV)	
				-0.17	(50-160 keV)	

Table IV:  $\beta^+$  and  $\beta^-$  transition results for MGT,  $\log(f_A t)$  and  $k, a, b, c$  shape parameter values using cluster wave functions from the text and PBWT shell model results (with their renormalized operators) from Towner and Hardy [24]. Expt. values for |MGT| and  $\log(f_A t)$  are from [25]. The slope parameter  $a$  for  $^{14}\text{C}$  is taken from Table II of [27] using the fitted value of  $a = -0.45(4)$  MeV $^{-1}$  in the energy range 100 – 160 keV. In electron rest mass units this becomes  $a = -0.23(2)$ . For the wider energy range  $a$  in electron rest mass units is  $-0.17$ .

For the special case of  $^{14}\text{O}(\beta^+)^{14}\text{N}(E = 0)$  the present work yields almost identical results to those of Towner and Hardy (who are the only ones to use renormalized couplings). The fit to the experimental data they obtained for the shape function using the PBWT [29] shell model basis shown in Fig. 2 has a slightly larger  $\chi^2$  than that obtained with the cluster model. The best fit to the data is obtained using a small change in  $\eta_+$  to 0.1085 which gives the dashed curve in Fig. 2 and corresponds to a  $\log(f_A t)$  of 7.51 and an MGT of 0.01373 for the  $^{14}\text{O}(\beta^+)$  transition. The sensitivity of the slopes  $a_+$  and  $a_-$  and the corresponding  $\log(f_A t)$  values to the values of  $\eta_+$  and  $\eta_-$  is shown in Fig. 3. The sensitivity of  $a_+$  to  $\eta_+$  is very much less than the sensitivity of  $a_-$  to  $\eta_-$  which in the cluster model is very sensitive.

The cluster  $^{14}\text{C}(\beta^-)^{14}\text{N}(E = 0)$  results are significantly different from those of Towner and Hardy as our wave-functions give the usual value of 9.04 for the  $\log(f_A t)$  when the slope parameter  $a_- = -0.231$  rather than the PBWT shell model  $\log(f_A t)$  of 8.72. We have been unable to exactly pin down the source of this difference. We note that the main thrust of the Towner and Hardy [24] investigation was to show that CVC could be satisfied accurately by using renormalized  $g$  factors and in the case of  $^{14}\text{O}(\beta^+)^{14}\text{N}_{gs}$  by adjusting the PBWT ground state of  $^{14}\text{N}$  they obtained a good description of the  $\beta^+$  shape function. We are concerned however that the Towner and Hardy calculations used a different wavefunction for  $^{14}\text{N}(E = 0)$  in the  $^{14}\text{C}$  decay from the one used in the  $^{14}\text{O}$  decay and recently Negret

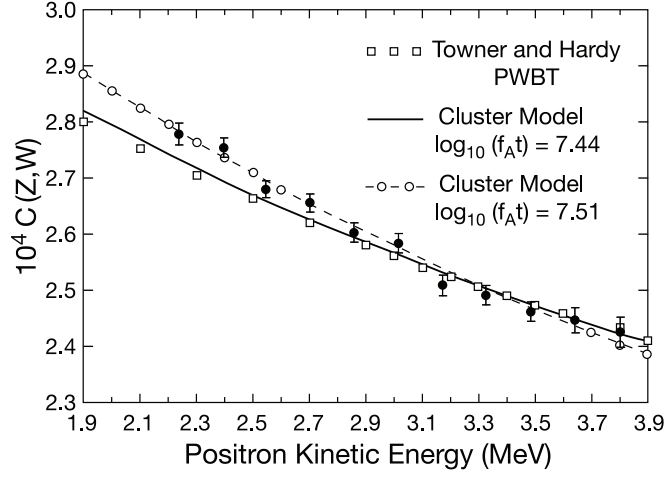


Figure 2: The shape function  $C(Z, W)$  for the  $^{14}\text{O}(\beta^+)^{14}\text{N}(E = 0)$  transition as a function of the positron kinetic energy (MeV). The data points with error bars are taken from [28] including corrections given in [24].

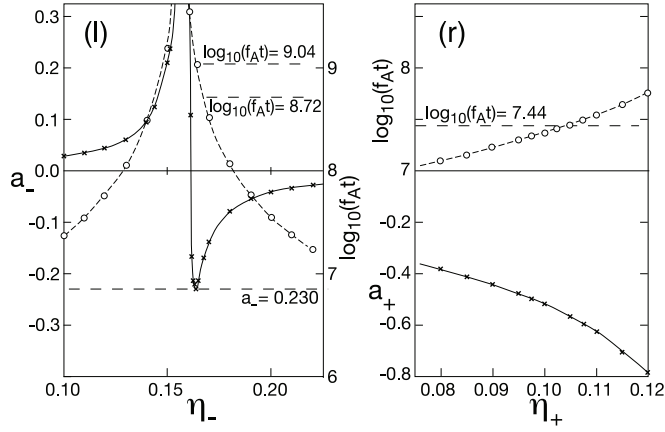


Figure 3: The values of  $a_-$  and  $a_+$  as a function of the respective initial state admixture coefficients denoted by  $\eta_-$  and  $\eta_+$  are shown by the continuous lines in the left and right side panels respectively. The corresponding values of the  $\log(f_A t)$  are shown by the dashed lines and for the  $^{14}\text{C}(\beta^-)$  case shows the strong variation in  $a_-$  and  $\log(f_A t)$  for small changes in  $\eta_-$  near 0.1635201.

*et al* [30] implied that Towner and Hardy had been able to account for the large asymmetry in the mirror  $\log(ft)$  values. Physically these mirror decays have the same final state in  $^{14}\text{N}$  and one cannot explain the large ratio of their MGT values by simply modifying the final state. Indeed Towner and Hardy did not claim [24] to have explained the large asymmetry in these mirror  $\log(ft)$  values; their focus was on reconciling the  $^{14}\text{O}$  shape-correction form factor with the M1 matrix element in  $^{14}\text{N}$ . As pointed out by early workers [2, 6] the symmetry breaking of these mirror transitions must arise from symmetry breaking interactions in the initial mirror states. Here we have specified that it is primarily the electromagnetic spin-orbit interaction in the initial states that causes the large asymmetry. It is also worth noting the opposite signs of the MGT values for these mirror transitions because the shape parameters do not come out correctly unless the mirror MGT values have opposite signs. This is most readily seen in Fig. 3 where the values of  $\eta_-$  between 0.14 and 0.15 can yield  $\log(f_A t)$  values between 8.5 and 9.4 with MGT > 0 but also yield a positive slope for  $a_-$  in complete contradiction to experimental data.

One of the difficulties remaining is the uncertainties in the experimental data [27] for the  $^{14}\text{C}(\beta^-)$  transition. In particular the value of the slope parameter depends strongly on the range of electron energies used as indicated in Table IV. Also the accuracy of the data apparently did not allow any information to be determined for the  $b$  or  $c$  coefficients in  $C(Z, W)$  so that a linear form  $C_L(Z, W) = (1 + a^*W)$  was used [27] to extract the effective slope  $a^*$  for each energy range. The value of  $a^* = -0.45(4) \text{ MeV}^{-1}$  for the 100-160 keV range is apparently believed to be the favored value for  $a^*$  in ref. [27]. In the case of the shell model calculations given in ref. [24] one can least squares fit the models shape functions over the energy range 100-160 keV with the linear form to obtain the effective  $a^*$ . We



find  $a^* = -0.203, -0.214$ , and  $-0.195$  for the three models labeled CK, PWBT, and MK respectively in Table III of Towner and Hardy. The corresponding values of  $a$  are  $-0.215, -.235$  and  $-.207$  which are only 6% – 9% higher than their respective  $a^*$  values. For the cluster model with  $\log(f_{At}) = 9.040$  and  $a = -0.231$  a linear fit to  $C(Z, W)$  gives  $a^* = -0.232$  because of the strong cancellation of the  $b$  and  $c$  terms in this case. The cluster case where the  $\log(f_{At}) = 8.718$  and  $a = -0.176$  when linearized yields  $a^* = -0.207$  which is consistent with the shell model values for  $a^*$ . If one accepts the best value for  $a^* = -0.23$  then the cluster wavefunction with  $\log(f_{At}) = 9.040$  is favored over all the other models. Apparently only more accurate data for the  $^{14}\text{C}(\beta^-)$  transition can provide the necessary information on  $a, b$  and  $c$ , or  $a^*$ .

The  $\beta^+$  decay of  $^{14}\text{O}$  leading to the  $^{14}\text{N}$  ( $E = 3.948$ ) state calculated here has a  $\log(f_{At})$  of 3.14 and an MGT of 2.119 in perfect agreement [25] with experiment. This is not discussed by anyone else except for Genz *et al* [11] who quoted that their model yielded a  $\log(f_{At})$  value of 2.87. In our case this latter transition was part of the fitting procedure whereas Genz *et al* did not include this transition as part of their fitting procedure. However we believe that calculations of this transition to the first excited  $1^+$  state in  $^{14}\text{N}$  are an additional restraint that all models should include.

The major reason that renormalized couplings were used is to understand whether the radiative ( $M1$ ) width of the  $^{14}\text{N}, T = 1$  state at  $E = 2.313$  is consistent with the model wave functions. Again the model states above yield a value of  $\Gamma_\gamma = 6.7\text{meV}$  but only if renormalized values of the isovector coupling constants from above are used thereby satisfying the conserved vector current requirements. The formulas are given by Garcia and Brown [26] in their eq.(25) in terms of the structure coefficients  $V_1$  and  $V_3$  given in [11] and with our renormalized magnetic couplings one needs in order to obtain  $\Gamma_\gamma$  that

$$|2^{-1/2}(\mu_p^* - \mu_n^*)V_1(\gamma) + g_{lp}^*V_3| = 0.256 \text{ n.m.} \quad (17)$$

with an associated error of 0.006 n.m. The value obtained here is 0.256 n.m. since the radiative width  $\Gamma_\gamma$  of 6.7(3) meV was part of our fitting procedure for the coefficients  $\alpha, \beta, \gamma, \xi_0$  and  $\eta_0$ . The recent work of Holt *et al* [10] also considered the recent experiments [30] that determined  $\text{BGT} = |\text{MGT}|^2/(2J_i + 1)$  values from the  $^{14}\text{N}$  ground state to excited states of  $^{14}\text{C}$  and  $^{14}\text{O}$  using the charge exchange reactions  $^{14}\text{N}(\text{d}^2, \text{He})^{14}\text{C}$  and  $^{14}\text{N}(^3\text{He}, \text{t})^{14}\text{O}$ . For the three final states in each of  $^{14}\text{C}$  and  $^{14}\text{O}$  labeled as  $0_1^+, 0_2^+$  we have wavefunctions as listed above and for the  $1_1^+$  states at 11.31 MeV and 11.24 MeV respectively we use a pure  $^3P_1$  two nucleon-hole in  $^{16}\text{O}$  as did Amos *et al* [13]. For the  $0_1^+$  states the beta decay data is more reliable and we already fitted their BGT values. For the  $0_2^+$  states we obtain  $\text{BGT} = 0.028$  for both these transitions which are similar to the very small values shown in Negret *et al* [30]. By using the definition in Holt *et al* for  $\text{B}(\text{GT})$  corresponding to the inverse transition (note that  $g_A$  is not included in their definition of  $\text{B}(\text{GT})$ ) we obtain for the  $^{14}\text{C}$  case that  $\text{B}(\text{GT}) = .071$  which appears to agree closely with the experimental value shown in their Fig. 2 for this transition. The Holt *et al* theory result for this transition is at least three times too large even with the modified tensor interaction included. The  $M1$  transition from the ground state of  $^{14}\text{C}$  to the  $1_1^+$  state at 11.3 MeV in  $^{14}\text{C}$  was first discussed in [13] as a way to obtain information on the  $^3P_0$  component in the ground state of  $^{14}\text{C}$ . The radiative width  $\Gamma_\gamma$  of this  $1_1^+$  state was measured [31] to be 6.8 meV by extrapolating the inelastic electron scattering from  $^{14}\text{C}$  data to the photon point. However such extrapolations can be quite inaccurate and it is more reliable for this  $1_1^+$  state to consider the BGT measurements of Negret *et al*. The calculated value of BGT to this state in the cluster model is  $\text{BGT} = 0.082$  or  $\text{B}(\text{GT}) = 0.069$  which appears to be only about 30% below the experimental value.

The situation for the transitions to  $J^P = 2^+$  states from the  $^{14}\text{N}$  ground state is beyond the scope of this work as it is too complicated for us to calculate with any accuracy in the cluster model since there are three  $2^+$  states observed in each mirror system with significant strength and only two  $2^+$  states with the simple two-hole structure in the  $^{16}\text{O}$  ground state. However the two simple states,  $^1D_2$  and  $^3P_2$ , will have a strong BGT only for the  $^1D_2$  because the initial state has over 98% in amplitude in the  $^3D_1$ . This means that the transitions to the three mixed  $2^+$  states observed will dominate the GT transitions as suggested by Aroua *et al* [9] and observed in the experiments of Negret *et al*.

## VI. ELASTIC AND INELASTIC M1 ELECTRON SCATTERING

In this section it is necessary to go beyond the admixture coefficients  $\alpha, \beta, \gamma, \xi_L, \eta_L$  and the angular momentum quantum numbers because the form factors for  $M1$  transitions involve momentum transfer ( $q$ ) dependence and four types of multipoles for each nucleon [32]. In the  $p^{-2}$  simple shell model it is customary to use a single p-shell harmonic oscillator wave function for each nucleon and this yields very simple spherical Bessel transforms for  $L = 0$  and  $L = 2$  amplitudes. These are given by Willey [32] as radial integrals over  $j_L(qr)$  and a unit normalized 1p radial density :

$$\langle j_0 \rangle_{1p,1p} = (1 - 2x/3)e^{-x}, \quad \langle j_2 \rangle_{1p,1p} = (2x/3)e^{-x} \quad (18)$$

in which  $x = q^2 b^2/4$  and  $b$  is the usual three dimensional harmonic oscillator length. In the case of  $^{15}\text{N}$  there is elastic electron scattering data available [33] for the  $M1$  form factor which we denote as  $F_T(q)$ . In the simple shell model this  $M1$  form factor involves a  $1p_{1/2}$  proton-hole description and the form factor is given by:

$$F_T(q) = K\mu q\{(1 - 2x/3) + 4x/9(g_{pl} + \mu_p)/\mu\}e^{-x}F_{SN}F_{c.m.} \quad (19)$$

in which  $F_{SN}$  is the nucleon size form factor and  $F_{c.m.}$  for harmonic oscillator states is a simple Gaussian so that  $e^{-x}F_{c.m.}$  is replaced by  $e^{-x(A-1)/A}$  with  $A = 15$ . In this work we use a simple dipole form for  $F_{SN} = (1 + .054675q^2)^{-2}$  corresponding to an rms radius of 0.81 fm. The observed magnetic moment  $\mu$  is  $-0.2831888$  n.m. and  $K = 2^{-1/2}\hbar c/(2\pi\text{Mc}^2 Z)$  has the value 0.02124 fm for  $Z = 7$ . The above formula has been checked for the shell model calculations with various  $b$  values and reproduces the results given in [33] using free nucleon  $g_{pl}, \mu_p$  values. As Singhal *et al* [33] point out the simple shell model calculations overestimate the peak value of  $|F_T|^2$  by 20-30% and underestimate it for  $q_{eff}$  beyond  $2.4 \text{ fm}^{-1}$  by large factors. Using our renormalized  $g_{pl}^*, \mu_p^*$  in place of the free values in (19) makes the shell model calculated  $|F_T|^2$  values even larger and makes no significant improvement in the large  $q$  behavior. Only by including configurations from the  $2p_{1/2}$  shell can the data be fitted [33] for all  $q$  values.

In the cluster model for  $^{15}\text{N}$  we expect the radial distribution of the proton-hole in the highly correlated tetrahedral alpha-like cluster  $^{16}\text{O}$  reference state to be considerably different from the  $1p$  independent particle radial wave function. Lacking a detailed theory for the many-body interactions it is convenient to use a simple expansion for the Fourier-Bessel densities (18) as a linear superposition of  $1p$  densities with different values of the oscillator parameter:

$$F_{c.m.}\langle j_0 \rangle_{1p,1p} = n^{-1} \sum_n (1 - 2x(n)/3) e^{-(A-1)x(n)/A}, \quad F_{c.m.}\langle j_2 \rangle_{1p,1p} = n^{-1} \sum_n 2x(n)/3 e^{-(A-1)x(n)/A} \quad (20)$$

in which  $x(n) = q^2 b_n^2/4$  and  $n$  is limited to four. The four values of  $b_n$  are found by fitting the  $M1$  elastic electron scattering data for  $^{15}\text{N}$  and  $^{14}\text{N}$ . For the  $^{14}\text{N}$  case we use the similar expressions to those used in [34]

$$F_{elT}(q) = qn^{-1} \sum_n e^{-(A-1)x(n)/A} (A_0 + A_1 x(n)) F_{SN} \quad (21)$$

$$F_{inT}(q) = qn^{-1} \sum_n e^{-(A-1)x(n)/A} (B_0 + B_1 x(n)) F_{SN} \quad (22)$$

with the  $A_i, B_i$  amplitudes being given in terms of the structure amplitudes in  $L$ - $S$  coupling as

$$A_0 = (2/3)^{1/2} \mu K, \quad A_1 = -(2/3)^{1/2} K \mu_d^* (W_1 - W_2)/3 \quad (23)$$

$$B_0 = -(3/2)^{1/2} K \{2^{-1/2}(\mu_p^* - \mu_n^*)V_1 + g_{lp}^* V_3\}, \quad B_1 = (2/3)^{1/2} K (\mu_p^* - \mu_n^*) (V_1 - V_2)/3 \quad (24)$$

in which  $\mu$  is the magnetic moment of  $^{14}\text{N}$  (calculated here to be the observed value of  $-0.2831888$  n.m.). The coefficients  $W_i$  are defined in Genz *et al* [11] and in our model they are:

$$W_1 = 2\alpha^2 - \gamma^2, \quad W_2 = -(4/5)^{1/2} \alpha \gamma + (27/10)^{1/2} \beta \gamma + \gamma^2 \quad (25)$$

in which the  $W_2$  differs from theirs by the coefficient of  $\gamma^2$  due to the deuteron-like hole in the same alpha-like cluster having the orbital angular momentum ( $L = 2$ ) entirely in the coordinate connecting the center of mass of the hole pair to the center of mass of the reference system. Note that the quadrupole moment  $Q$  of  $^{14}\text{N}$  is given here by:  $Q = \langle r^2 \rangle/5 \{ (16/5)^{1/2} \alpha \gamma - \beta^2 + \gamma^2 \}$  which differs from the expression for  $Q$  in Genz *et al* only by our using unity instead of  $7/10$  as we also used in (17). The coefficients  $V_i$  are identical to those in Genz *et al*;  $V_1 = -2^{1/2}(\xi_0 \alpha - \eta_0 \beta/3^{1/2})$ ,  $V_2 = -((2/5)^{1/2} \xi_0 \gamma + 6^{-1/2} \eta_0 \beta + (9/20)^{1/2} \eta_0 \gamma)$  and  $V_3 = -(2/3)^{1/2} \xi_0 \beta + (2/9)^{1/2} \eta_0 \alpha - (5/18)^{1/2} \eta_0 \gamma$  which when used in (17) yield with the wave functions in section V the result of 0.256 n.m. as needed for the radiative width to be 6.7 meV. The value of the radiative width being 6.7 meV in the model used here is independent of up to 30% variations in the value of  $\eta_0$ . In this model the value of  $\eta_0$  must satisfy  $\eta_- > \eta_0 > \eta_+$  and in general  $\eta_0$  lies approximately half-way between  $\eta_-$  and  $\eta_+$ . Since the model is close to  $L$ - $S$  coupling for all ground state wave functions and all the  $\eta$ 's are less than  $+0.2$  to yield good descriptions of the beta-decay data to the ground state of  $^{14}\text{N}$  it follows that the cluster picture with consistent renormalized operators  $g^*$  is in full agreement with the requirements of CVC.

The form factor for elastic electron  $M1$  scattering has been fitted using the set of four  $b_n$  values (all in fm units)  $b_1 = 2.85, b_2 = 1.95, b_3 = 1.82$  and  $b_4 = 1.32$  and shows excellent agreement with the experimental data [34] in Fig. 4. The *same* set of oscillator lengths is used to calculate the elastic electron  $M1$  scattering form factor for

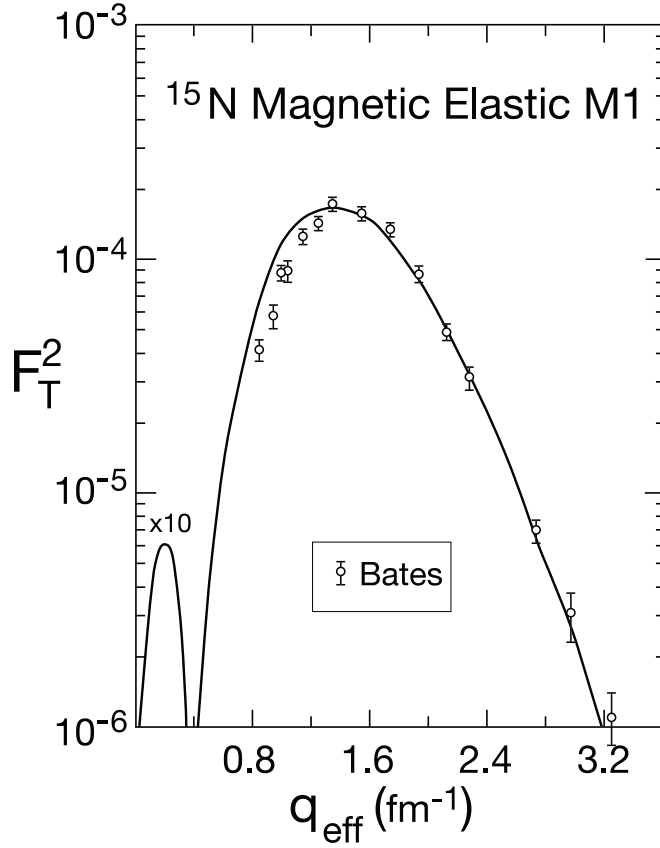


Figure 4: The transverse magnetic elastic form factor for  $^{15}\text{N}$  calculated using the cluster model parameters described in section VI and shown by the continuous curve is compared to the Bates data [33].

$^{14}\text{N}$  and is compared with the experimental data in the lower panel of Fig. 5. The fit is of the same quality as that shown for  $^{15}\text{N}$  in Fig. 4. The consistency of the model for  $A = 14$  and  $A = 15$  gives some credence to the use of the cluster modified  $1p$  density given in (20) above. The rms proton-hole radius  $\text{rmsph}$  is calculated using  $\text{rmsph} = \{5(A-1)/8(b_1^2 + b_2^2 + b_3^2 + b_4^2)/A + (0.81)^2\}^{1/2} = 3.250 \text{ fm}$  ( $A = 15$ ), or  $3.242 \text{ fm}$  ( $A = 14$ ) and it is the  $A = 14$  value we need for the calculation of  $Q$  as defined above. Using the values of  $\alpha, \beta$ , and  $\gamma$  given in section V above we find that  $Q = 20.18 \text{ mb}$  that agrees well with the recent experimental values of  $19.3(8) \text{ mb}$  and  $20.01(10) \text{ mb}$  from the recent compilation [35]. A small correction to the calculated value of  $Q$  arises because of the small value of  $P_D$  needed to fit the magnetic moment of  $^{14}\text{N}$ . The estimate of this correction is uncertain but one obtains a value of  $-Q_d^* \gamma^2/10$  in which  $Q_d^*$  is the quadrupole moment of the bound deuteron in  $^{16}\text{O}$ . We anticipate that  $Q_d^*$  is most likely less than the  $Q_d$  of the free deuteron ( $+2.86 \text{ mb}$ ) so that the correction is expected to lie between  $0$  and  $-0.3 \text{ mb}$  where the minus sign is because of the hole nature of the deuteron. Thus our estimate of  $Q$  for  $^{14}\text{N}$  is  $20.03(15) \text{ mb}$  where the error is comparable to the error in the experimental value for  $Q$ . It is interesting to note that the early value of  $Q$  given in 1955 by Sherr *et al* was about  $7 \text{ mb}$  and it grew steadily over the next 38 years to the most accurate value of  $20.01(10) \text{ mb}$  in 1993.

It is also possible to use the simple relation used in [36] between the  $^{16}\text{O}$  charge radius and the  $^{15}\text{N}$  charge radius for our model state for the ground state of  $^{15}\text{N}$ :

$$\langle r^2 \rangle_{ch}^{1/2} (^{15}\text{N}) = \{8/7 \langle r^2 \rangle_{ch} (^{16}\text{O}) - (\text{rmsph})^2/7\}^{1/2} = \{8/7(2.71)^2 - (3.25)^2/7\}^{1/2} = 2.62 \text{ fm} \quad (26)$$

which agrees very well with the result given for the charge radius of  $^{15}\text{N}$  in [36].

The inelastic electron scattering from  $^{14}\text{N}$  leading to the first excited state at  $E = 2.313 \text{ MeV}$  shows much more deviation from the shell model calculations than the elastic data does and because this transition involves a change of isospin from  $T = 0$  to  $T = 1$  it has the connection to beta-decay as indicated in earlier work concerned with CVC. As we noted earlier for the beta-decay  $C(Z, W)$  shapes we need a knowledge of the average oscillator length squared which we denote by  $b_c^2$  and which we can assume should be the average of a set of  $b_n^2$  that fit the inelastic electron scattering form factor. Our fit to the data using (all in fm units)  $b_1 = 3.512, b_2 = 2.12, b_3 = 1.50$ , and  $b_4 = 1.09$

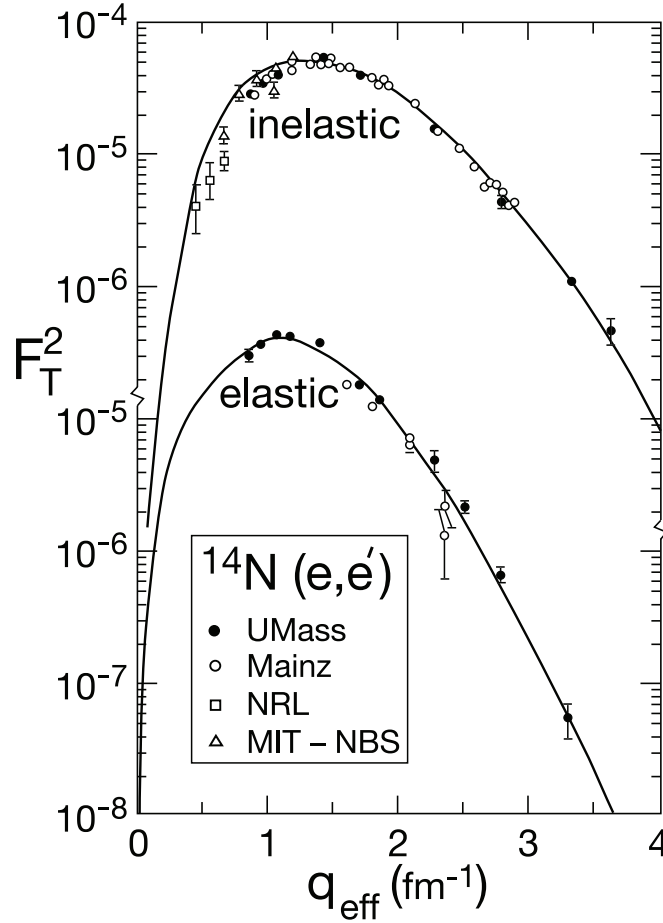


Figure 5: Measured elastic and inelastic  $M1$  form factors for the ground state and 2.313 MeV transition in  $^{14}\text{N}$  taken from [34] (and references therein) are compared with cluster model calculations using the parameters defined in section VI.

describes the data very well including the large  $q$  values. The value of  $b_c$  from  $b_c^2 = (b_1^2 + b_2^2 + b_3^2 + b_4^2)/4$  is 2.25 fm which was used to calculate the shapes for the beta decays discussed in section V above.

## VII. GAMMA DECAYS IN $^{14}\text{N}$

In most of the work on beta decay in the  $A = 14$  system only the  $M1$  decay of the first excited state has been calculated. In this section we focus on the gamma decay of the  $J^P = 1^+$  at  $E = 3.948$  MeV excitation. The gamma decay to the ground state involves two multipoles corresponding to  $E2$  and  $M1$ . The wave functions in section V gives the values of 0.0026 eV and 0.00042 eV for the radiative widths for  $E2$  and  $M1$  respectively which agree well with the corresponding experimental values 0.003 eV and 0.0004 eV from the TUNL compilation [37]. The gamma decay of the 3.948 MeV state to the  $T = 1, 0^+$  state (2.313 MeV) is the almost completely dominant decay mode and is pure  $M1$ . The wave functions in section V give a radiative width of 0.155 eV which is in good agreement with the experimental value of 0.091(30) eV. Overall the cluster model calculated gamma decay widths of the 3.948 MeV state are in satisfactory agreement with experiment but much of the gamma decay data in  $^{14}\text{N}$  is quite old and new measurements could provide more accuracy on the transitions between the low-lying positive parity states.

## VIII. DISCUSSION AND CONCLUSIONS

The results given in the preceding sections represent a convincing argument that the standard simple shell model is not an optimal starting point for describing the ground state of  $^{16}\text{O}$  which, in turn, means it is not an optimal starting point for describing the low-lying states of  $A = 15$  and  $A = 14$  nuclei. In particular the cluster model need for the  $0^+$

ground states of  $A = 14$  nuclei with  $T = 1$  to be more than 97% in the  $^1S_0$  configuration (corresponding in  $j$ - $j$  coupling to 1/3 probability for the  $p_{1/2}^{-2}$  component and 2/3 probability for the  $p_{3/2}^{-2}$  component) contradicts the expectations of the strong spin-orbit  $j$ - $j$  shell model ideology as pointed out by Talmi [8]. More important however is that the strong correlations leading to alpha-like clustering with  $T_d$  point group symmetry is that  $^{16}\text{O}$  is a tetrahedrally deformed nucleus and not spherical, as all shell model calculations use as a basic starting point. It is strongly deformed as the ground state rotational band [19] with the sequence  $0^+, 3^-, 4^+, 6^+$ , has strongly enhanced  $BE3, BE4$  values for the transitions from the  $3^-, 4^+$  states respectively to the ground state that are typical of a simple tetrahedral rotor model. There is no quadrupole deformation in the tetrahedral model of the ground state rotational band of  $^{16}\text{O}$  as it violates the boson symmetry for four identical alpha-like clusters (tetrons?). *We believe for  $N = Z$  even-even nuclei that the clustering dynamics of  $(N + Z)/4$  identical tetron clusters determines the various multipole deformations with  $L > 1$  for each nucleus.*

We note that the next alpha-like cluster system to not have any quadrupole deformation in the lowest energy intrinsic state is  $^{40}\text{Ca}$  as it appears to also be tetrahedrally deformed rather than spherical. In particular the lowest lying excited states are  $0^+(E = 3.35 \text{ MeV})$  and  $3^-(E = 3.74 \text{ MeV})$  with the  $BE3$  value being 31 (W.U). Surprisingly at first is the fact that the binding energy of the last neutron (15.643 MeV) in  $^{40}\text{Ca}$  is almost identical to the binding energy (15.664 MeV) of the last neutron in  $^{16}\text{O}$ . The difference is 21 keV and we do not for one moment believe this is an accidental degeneracy. We conjecture that the last neutron taken from one of the outermost tetron clusters will have three neighboring tetron clusters which are in the same close packed configuration as the four tetrons in  $^{16}\text{O}$ . The remaining six tetrons in a tetrahedral  $^{40}\text{Ca}$  are all spatially removed from the four containing the last neutron so that there is almost no interaction between the six spectators and the neutron being taken out. In short the similarity in the neutron binding energy in the tetrahedral model arises because the weakest bound neutron interacts only with the nearest neighbor clusters which is the same in  $^{16}\text{O}$  and  $^{40}\text{Ca}$ . Of course this does not happen for the binding energy of the last proton because it sees the long range Coulomb interaction from all the spectators.

In summary the new solution to the  $A = 14$  beta-decay puzzle uses a highly correlated model in which the unperturbed LSJT basis has a simple orbital angular momentum selection rule which forbids the ground state GT transitions in the mirror  $\beta$ -decays. This rule arises in this model because the  $L = 2, S = 1, J = 1, T = 0$  state is the most appropriate assignment for  $^{14}\text{N}_{\text{gs}}$  and the initial mirror  $T = 1$  ground states in the  $p^{-2}$  basis space (even with mixing) has no  $L = 2$  component. Only by mixing in  $L = 1$  and  $L = 0$  states in the  $T = 0$  sector can the mirror ground states have non-zero MGT elements. In the shell model the use of reasonably strong tensor and spin-orbit interactions causes the  $L = 2$  state in  $^{14}\text{N}_{\text{gs}}$  to become significantly mixed with the  $L = 0$  and  $L = 1$  basis states which has to be fine tuned to cancel the MGT elements when the  $T = 1, L = 0$  and  $L = 1$  states in  $^{14}\text{C}$  and  $^{14}\text{O}$  are automatically strongly mixed by the strong spin-orbit interaction. The nature of the reference  $^{16}\text{O}$  state in this alternative model leads to the expectation of a strong suppression of the nuclear spin-dependent mixing interactions which is why the model has weak mixing between the LSJ states for both  $T = 0$  and  $T = 1$  states and changes the infinite life of  $^{14}\text{C}$  into a long lifetime. The model explains the large asymmetry between the mirror  $\beta$ -decays because of the interference between the nuclear and electromagnetic spin-orbit mixing interactions with the latter term having opposite sign for the state in  $^{14}\text{O}$  to that in  $^{14}\text{C}$ . The MGT elements for these two transitions are indeed very different in magnitude with this model and also their MGT elements have opposite sign in order to obtain the observed negative values for the  $a_+, a_-$  shape slope parameters. Hopefully the detailed results provided in this work on the  $\beta$ -decays in  $A = 14$  nuclei will be helpful to future investigators in their search for a more complete description of the structure of p-shell nuclei.

### Acknowledgments

My colleagues at Florida State University, in particular K. Kemper, A. Volya, V. Abramkina, and S. Tabor are greatly acknowledged. Special thanks for several discussions are due to J. C. Hardy, B. A. Brown, A. Richter and D. J. Millener.

- 
- [1] D.R. Inglis, Rev. Mod. Phys. **25**, 390 (1953).
  - [2] B. Jancovich and I Talmi, Phys. Rev. **95**, 289 (1954).
  - [3] W.M. Visscher and R.A. Ferrell, Phys.Rev. **107**, 781 (1957).
  - [4] L. Zamick, D.C. Zheng and M.O. Fayache, Phys. Rev. **C51**, 1253 (1995).
  - [5] M.S. Fayache, L. Zamick and H. Muther, Phys. Rev. **C60**, 067305 (1999)
  - [6] R. Sherr *et al*, Phys.Rev. **100**, 945 (1955).
  - [7] J.P. Elliott, Phil. Mag. **1**, 503 (1956).

- [8] I. Talmi, Phys. Rev. **C39**, 284 (1989).
- [9] S. Aroua *et al*, Nucl.Phys. **A720**, 71 (2003).
- [10] J.W. Holt *et al*, Phys. Rev. Lett. **100**, 062501 (2008), (see also J.W. Holt, N. Kaiser, and W. Weise, Phys. Rev. **C79**, 054331 (2009)).
- [11] H.Genz *et al*, Z. Phys. **A341**, 9 (1991).
- [12] R.L. Huffman *et al*, Phys.Rev. **C35**, 1 (1987).
- [13] K. Amos, D. Koetsier and D. Kurath, Phys. Rev. **C40**, 374 (1989).
- [14] J.A. Wheeler, Phys. Rev. **52**, 1083 (1937).
- [15] D.M. Dennison, Phys. Rev. **57**, 454 (1940), *ibid* **96**, 374 (1954).
- [16] S.L. Kameny, Phys. Rev. **103**, 358 (1956).
- [17] M. Freer, Rep. Prog. Phys. **70**, 2149 (2007).
- [18] D. Robson, Nucl. Phys. **A308**, 381 (1978).
- [19] D. Robson, Prog. in Part. and Nucl. Phys. **8**, 257 (1982).
- [20] D. Robson, Phys. Rev. **D35**, 1018,1029 (1987) and Phys. Rev. **C61**, 015202 (1999).
- [21] C. Long and D. Robson, Nucl. Phys. **A474**, 627 (1987).
- [22] J.S. Mc Carthy, I. Sick and R.R. Whitney, Phys. Rev. **C15**, 1396 (1977), (see also R. Arnold *et al*, Phys. Rev. Lett. **40**, 1029 (1978)).
- [23] J.A. Nolen and J.P. Schiffer, Ann. Rev. Nucl. Sci. **19**, 471 (1969).
- [24] I.S. Towner and J.C. Hardy, Phys. Rev. **C72**, 055501 (2005) and J. C. Hardy, private communication (2010).
- [25] W.T. Chou, E.K. Warburton and B.A. Brown, Phys. Rev. **C47**, 163 (1993).
- [26] A. Garcia and B.A. Brown, Phys. Rev. **C52**, 3416 (1995).
- [27] F.E. Wietfeldt *et al*, Phys. Rev. **C52**, 1028 (1995).
- [28] G.S. Sidhu and J.B. Gerhart, Phys. Rev. **148**, 1024 (1966), (see also ref. [24] for corrections to C(Z,W)).
- [29] E.K. Warburton and B.A. Brown, Phys. Rev. **C46**, 923 (1992).
- [30] A. Negret *et al*, Phys. Rev. Lett. **97**, 062502 (2006).
- [31] H. Crannell *et al*, Nucl. Phys. **A278**, 253 (1977).
- [32] R.S. Willey, Nucl. Phys. **40**, 529 (1963).
- [33] R.P. Singhal *et al*, Phys. Rev. **C28**, 513 (1983).
- [34] R.L. Huffman *et al*, Phys. Lett. **139B**, 249 (1984).
- [35] N.J. Stone, At. Data Nucl. Data Tables **90**, 75 (2005).
- [36] J.W. de Vries *et al*, Phys. Lett. **205B**, 22 (1988).
- [37] F. Ajzenburg-Selove, Nucl. Phys. **A523**, Table 2 (1991).